Forecasting the demand for currency

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This article examines a number of methods for forecasting the demand for currency in circulation, and explains how these are used by the Currency Department of the Bank.

I Introduction

Over the last couple of years, the Reserve Bank has found it difficult to forecast accurately the amount of notes and coins demanded by registered banks. As a result a team of staff developed a series of models which would assist in forecasting currency trends. This paper summarises the results of that study, and presents a number of equations which are assessed for their forecasting ability. Currency demand equations based on economic theory and time series ARIMA models are used.

The rest of the article is organised as follows. Section II describes the background to the currency forecasting project. Section III summaries the modelling techniques that were used. Section IV briefly describes the data used in the estimation. Section V presents the results of the modelling, and finally Section VI provides a summary and conclusions.

II Background

As a result of a relatively buoyant economy in recent years, particularly in the retail and tourism sectors, we have seen a significant increase in the cash holdings demanded by the general public. This increase is reflected in figure 1 overleaf. However, over the last two years the extent of this increase in the demand for currency has not been fully anticipated by the Bank. As a consequence of this underestimation, actual currency expenses incurred by the Bank over this period exceeded budgeted estimates.

At first glance the rising demand for currency seems rather surprising, since there have been significant developments in the application of electronic technology in the retail payments area over the last few years. These include strong growth in the use of ATMs (Automatic Teller Machines) and EFTPOS (Electronic Funds Transfer at Point Of Sale) (see figure 2). EFTPOS is currently experiencing an annual average growth rate of about 60 percent. The use of ATMs follows a similar, but slightly less spectacular pattern, with an annual average growth rate of around 40 percent. We would expect wider use of electronic technology to result in the demand for currency being reduced. However, ATMs and EFTPOS facilities have made cash more accessible to the public. As a result, many transactions that were previously made by cheque (because cash was not readily available, say on weekends) can now be made with cash.

In addition, the cost of carrying out many bank transactions has increased recently as banks have introduced more comprehensive pricing of transactions services. These bank transactions charges may create an incentive for customers to make small payments by cash to avoid the bank transaction charges, and probably also encourage customers to make fewer, larger, cash withdrawals. Both these tendencies would increase the amount of currency demanded. Therefore, it is unclear whether the overall demand for cash should have increased or decreased as a result of recent technology innovations. The statistics to date suggest the former rather than the latter to be the case.

Further innovation is expected with the introduction of pre-paid ‘smart’ cards, which some financial institutions are currently trialling. These are stored value cards which use computer chips to record an amount of credit and the amount spent in using up that credit. These cards can be reloaded at will, and are designed as a replacement for small cash transactions. It is expected that within the next couple of years the upward trend for currency in circulation will ‘level off’ and eventually decline as smart cards become more widely used in the community.

III Modelling techniques

Most of the previous modelling work on money aggregates in the Bank had focused on their potential use in monetary policy, either as policy targets or as indicators. As a result, the previous empirical work had concentrated on testing the stability of money aggregate demand functions, where the money aggregates were much broader than just currency in circulation (like M1, M2 and M3), and the approach was to identify structural relationships based on economic theory. However, since the purpose of the current study was purely to forecast future movements of the currency stock, we examined alternative forecasting tech-

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1 See Ledingham (1994) for further details on pre-paid cards.
Figure 1
The value of currency in circulation

Source: RBNZ Currency Department

Figure 2
Financial payments by type
(number of transactions)

Source: New Zealand Bankers’ Association
niques that do not rely on a structural relationship between money and other nominal or real variables. In addition to estimating a currency demand function, we also estimated ARIMA models. Both approaches are described in more detail below.

(i) Currency demand function

The first approach used to forecast currency was a standard money demand function based on the transactions demand for money and portfolio demand for money. This can be expressed as

$$M^d_t = f(P_t, Y_t, R_t)$$

where the amount of money demanded in period $t$ depends on the price level $P_t$, the level of real output $Y_t$, and the opportunity cost $R_t$ of holding cash. If we assume this function has a log-linear form, this can be written as

$$m_t = \alpha + \beta_1 p_t + \beta_2 y_t + \beta_3 R_t$$

where lowercase letters represent logarithmic variables.

Earlier studies of the relationship between money demand (in terms of money aggregates) and changes in nominal and real sector variables provided a less than encouraging backdrop. Although this earlier work examined a wide variety of broad and narrow aggregates, most of the studies failed to find a stable demand function (eg, Siklos, 1995). This is consistent with studies carried out in other countries, which suggest that new technology and financial innovation have made the parameters in money aggregate demand functions unstable (Goldfeld and Sichel, 1990; Cuthbertson, 1991).

Surprisingly, there has been little work at the Bank which has used money demand functions for estimating the demand for currency. One exception is the study by Greville (1989), who estimated a money demand function for notes. Greville found that the estimated equations were only successful if a share price index was one of the variables used to explain the demand for money.

(ii) ARIMA models

The second approach used to model the demand for currency was the Auto-Regressive Integrated Moving Average (ARIMA) technique developed by Box and Jenkins (1970). The models constructed using this approach are independent of any particular economic theory, and the forecasts from the models are based purely on the past behaviour of the series of interest. The ARIMA technique uses the properties of a ‘stationary’ time series to forecast its future movement. In simple terms, a stationary time series is one which tends to return to its mean value after an increase or decrease. A non-stationary series does not have this tendency, and will only change if it receives an external shock. If the series is non-stationary, then it is first differenced to make it stationary, and the auto-regressive / moving average modelling is carried out on the differenced series.

IV Data

Figure 3 shows the year-over-year growth in currency issued into circulation (net of currency repatriated to the Reserve Bank) from 1983 onwards. The graph shows that annual growth in currency has had some large spikes and dips in recent years. The large dip in 1983 may be linked to the introduction of the wage and price freeze. The spike in 1987 could be related to either the financial liberalisation and the share market boom or the introduction of GST. The cyclical upturn in currency growth since mid-1992 is also apparent in the graph.

For the currency demand function several series were tested to represent each of the explanatory variables. In most money demand equations, prices and real output are used to represent the transactions demand for money. However, Mankiw and Summers (1986) argue that GDP is a poor indicator of the expenditure made with narrow money aggregates. As a result, we also tested variables such as consumption and retail sales to represent the value of transactions.4

Similarly, to represent the opportunity cost of holding cash, in addition to interest rate variables, we have also used surveyed inflation expectations, as described in Siklos (1995).

As mentioned above, technology innovations may have had a strong influence on the demand for cash in recent years. Unfortunately it has been difficult to find variables to represent these developments satisfactorily. Although we have obtained data on ATM terminals from the New Zealand Banker’s Association, the series is only available for recent periods. We have also had problems gathering EFTPOS data.5 Obtaining data on payments made with

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3 See Pindyck and Rubinfeld (1981) for a more technical description of ARIMA modelling techniques.

4 See also Greville (1989)

5 There are two separate EFTPOS networks in New Zealand. One network is run by ANZ / Postbank, and the other by Electronic Transactions Services Limited (ETSL) on behalf of other financial institutions.
these technologies remains a priority for any future analysis of currency demand in New Zealand.

V Forecasting results

The forecasting performance of the most satisfactory currency demand function and ARIMA models are presented below. The currency demand function is estimated using an error correction representation (ECM).\(^6\) Two ARIMA models appeared to have very similar properties, so we have reported the results of both. The first ARIMA model is a Seasonal Moving Average model of order 1 (ARIMA 1). The other ARIMA is a Seasonal Autoregressive Model of order 2 (ARIMA 2). We have examined the forecast errors both in-sample and out-of-sample.\(^7\) Full details of the models are presented in Appendix 1.

(i) In-sample forecasting

The ability of the three models to measure the level of currency in circulation is measured in the table below using percentage root mean squared errors (PRMSE) over different time periods. These figures represent the average forecast errors made by each model in any time period. Clearly, over the last decade the error correction model has outperformed the two ARIMA models. This is not surprising since over such a long horizon the structural relationships in the ECM should influence the data, and the model should dominate the simple extrapolative ARIMAs. However, even over the last six quarters of the estimation period the error correction model still outperforms the ARIMAs. There is also no evidence that the structural relationship has broken down in the last year, since the errors over this period are actually lower than over the 10-year period as a whole.

<table>
<thead>
<tr>
<th>Period</th>
<th>Money demand function (ECM)</th>
<th>ARIMA 1 (SMA(1))</th>
<th>ARIMA 2 (SAR(2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>June 1985 - June 1995</td>
<td>1.71%</td>
<td>2.87%</td>
<td>2.85%</td>
</tr>
<tr>
<td>March 1994 - June 1995</td>
<td>0.92%</td>
<td>2.12%</td>
<td>2.22%</td>
</tr>
</tbody>
</table>

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\(^6\) An error correction representation assumes that a long-run relationship exists between a set of variables. Divergences from this long-run relationship are used to explain movements in one of the variables. For example, if the relationship between money \(m\), prices \(p\) and output \(y\) is \(m = \beta_1 p + \beta_2 y\), then the error correction representation is \(\Delta m_t = \alpha (m_t - \beta_1 p_t - \beta_2 y_t)\), where the expression in the brackets represents divergences from the long-run relationship.

\(^7\) In-sample forecasting refers to the models’ predictions over the time interval used to estimate the models’ parameters. Out-of-sample forecasting refers to the models’ predictions after this period.
To test whether the difference in the models’ forecasting ability is statistically significant we used the sign test developed by Diebold and Mariano (1995). These results confirmed that the difference between the ECM forecasts and the ARIMA forecasts is statistically significant, but the difference between the two ARIMAs is not.8

In summary, therefore, in-sample forecasts of the error correction model out-performed the forecasts of the ARIMA models.

(ii) Out-of-sample forecasting

Although the in-sample results described above are interesting, the main focus of this paper is to find a model that forecasts accurately out-of-sample, especially over a 1-year horizon. To examine the out-of-sample forecasting properties of the three models, we reduced the estimation period back to March 1994, and compared the predicted values from each model with the actual values over the period June 1994 - June 1995. As an alternative baseline, we also examined the accuracy of the forecasts for this period, using the then established method, which were made in June 1994. That method involved a more judgmental approach which took account mainly of projected inflation and economic growth rates, and recent currency trends. The Percentage Root Mean Squared Errors for each of these models are shown below.

<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Existing method</td>
<td>3.42%</td>
</tr>
<tr>
<td>Money demand function</td>
<td>2.62%</td>
</tr>
<tr>
<td>ARIMA 1</td>
<td>1.25%</td>
</tr>
<tr>
<td>ARIMA 2</td>
<td>1.91%</td>
</tr>
</tbody>
</table>

All of the estimated models out-perform the existing method over the period tested. As the PRMSEs above demonstrate, the error correction model’s out-of-sample forecasts over this period are inferior to the forecasts from the two ARIMAs. Since the currency demand function’s in-sample forecasts out-perform the ones from the ARIMA equations, this suggests that the estimated parameters in the demand function are strongly influenced by the observations in this period. The moving average model (ARIMA1) also appears to out-perform the autoregressive one (ARIMA2).

VI Summary

Forecasting the demand for currency in New Zealand has become increasingly difficult over the last few years with ongoing innovation in the financial sector. As a result, in mid-1995 a team of staff was established, with the objective of developing a model which would assist in predicting the demand for currency one year ahead. In this paper, we have compared the forecasting performances of three models which may be useful for this purpose - a money demand equation, a seasonal moving average ARIMA model, and a seasonal autoregressive model. The money demand model had better in-sample forecasts than the two ARIMAs. However, this finding was reversed in the out-of-sample forecasts.

There are a number of other reasons why an ARIMA model may be more suitable for the Currency department’s forecasting than the structural demand function. Firstly, it is highly likely that innovations in payments technology, particularly the introduction of ‘smart’ cards, will have a strong impact on the demand for currency in the long term. As a result, a money demand equation that does not allow satisfactorily for innovation will be significantly mis-specified. In addition, the structural relationships in an error correction model will only have an impact on the economy in the long term. For short-term horizons, such as one year, it is likely that a simple extrapolative model like an ARIMA will perform better while a structural model may still be useful for longer-term currency forecasting. This will require more development to take account adequately of the effect of financial innovation.

For the time being the most suitable model for forecasting one year ahead currency requirements papers is the moving average equation. Since the project was completed this model has been incorporated into the Bank’s forecasting process and, as the graph below demonstrates, the model’s forecasts have provided useful information about movements in currency demand.

8 Results are available from the authors on request.
Appendix 1  
Final models

(i) Money demand function

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>t-ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>log(MMNC)[-1]</td>
<td>-0.6235</td>
<td>-5.303</td>
</tr>
<tr>
<td>log(PCPI)[-1]</td>
<td>0.6235</td>
<td>5.303</td>
</tr>
<tr>
<td>log(NCP_Z)[-1]</td>
<td>0.2074</td>
<td>2.727</td>
</tr>
<tr>
<td>ENIET100[-1]</td>
<td>-0.3038</td>
<td>-2.743</td>
</tr>
<tr>
<td>DUMM87</td>
<td>0.0406</td>
<td>2.137</td>
</tr>
<tr>
<td>DUMM93</td>
<td>0.0307</td>
<td>2.761</td>
</tr>
<tr>
<td>Δlog(PCPI)</td>
<td>1.3817</td>
<td>4.769</td>
</tr>
<tr>
<td>Δlog(NCP_Z)</td>
<td>0.4255</td>
<td>2.943</td>
</tr>
<tr>
<td>Q1</td>
<td>-0.0395</td>
<td>-3.821</td>
</tr>
<tr>
<td>Q2</td>
<td>-0.0470</td>
<td>-5.172</td>
</tr>
<tr>
<td>Q3</td>
<td>-0.0697</td>
<td>-8.452</td>
</tr>
<tr>
<td>Constant</td>
<td>-1.5324</td>
<td>-2.388</td>
</tr>
</tbody>
</table>

Test for cointegration

The t-statistic on the error correction coefficient (-5.303) exceeds the critical values in Wong (1993).

Testing theoretical restriction

Unit price elasticity $F(1,36) = 0.0788 (0.7806)$

Diagnostic statistics

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Value (p-value)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Durbin-Watson</td>
<td>2.208 (0.093)</td>
</tr>
<tr>
<td>Q(12-0)</td>
<td>18.816 (0.636)</td>
</tr>
<tr>
<td>ARCH 4 F(4,29)</td>
<td>0.643 (0.085)</td>
</tr>
<tr>
<td>Normality Chi²(2)</td>
<td>4.939 (0.076)</td>
</tr>
<tr>
<td>RESET F(1,36)</td>
<td>3.328 (0.076)</td>
</tr>
</tbody>
</table>

(ii) ARIMA models

1. Seasonal Moving Average model SMA(1)

Sample period 1984:01 - 1995:02

Standard error of the estimate $0.0288$

\[ y_t = e_t - 0.75893e_{t-4} \]

\[ t\text{-ratio: } (-7.07) \]

where $y_t = \Delta \Delta \log(\text{MMNC})$
Autocorrelation check of residuals

To lag  | Chi square statistic   | Degrees of freedom
--------|------------------------|---------------------
       6 | 3.65 (0.601)           | 5                   
       12| 5.73 (0.891)           | 11                  
       18| 13.48 (0.703)          | 17                  
       24| 16.79 (0.819)          | 23                  

2. **Seasonal Autoregressive Model SAR(2)**

Sample period 1984:01 - 1995:02
Standard error of the estimate 0.0290

\[ y_t = -0.6992y_{t-4} - 0.33178y_{t-8} + e_t \]

t-ratios: (-4.52) (-2.05)

where \( y_t = \Delta_4 \Delta \log(MMNC) \)

Autocorrelation check of residuals

To lag  | Chi-square statistic  | Degrees of freedom
--------|-----------------------|---------------------
       6 | 3.51 (0.476)          | 4                   
       12| 3.92 (0.951)          | 10                  
       18| 11.62 (0.770)         | 16                  
       24| 16.84 (0.772)         | 22                  

**Appendix 2**

**List of variables**

<table>
<thead>
<tr>
<th>Notes and coins in circulation</th>
<th>MMNC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumers Price Index</td>
<td>PCPI</td>
</tr>
<tr>
<td>Real private consumption (s.a.)</td>
<td>NCP_Z</td>
</tr>
<tr>
<td>National Bank inflation expectations</td>
<td>ENIET100</td>
</tr>
<tr>
<td>- next 12 months</td>
<td></td>
</tr>
<tr>
<td>GST introduction and increase</td>
<td>DUMM87,</td>
</tr>
<tr>
<td>dummies</td>
<td>DUMM93</td>
</tr>
<tr>
<td>Seasonal dummies</td>
<td>Q1, Q2, Q3</td>
</tr>
</tbody>
</table>

\( \Delta \) denotes first differencing \((x_t - x_{t-1})\) and \( \Delta_4 \) denotes fourth differencing \((x_t - x_{t-4})\).

**References**


