A model for interest rates near the zero lower bound:

An overview and discussion

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Leo Krippner

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Summary

Operating monetary policy when interest rates are already at or near zero comes with many challenges, as many countries have discovered in recent years. One aspect is that, if effective easing beyond a zero policy rate is desired, the policy rate constrained at zero will no longer conveniently summarise the stance of monetary policy and its typical transmission into the yield curve (longer-maturity interest rates) and the economy.

In this note, I show how a framework for representing yield curve data in a zero lower bound (ZLB) environment can still allow monetary policy to be conveniently summarised in terms of an effective policy rate. The framework I present essentially removes the effect that the option to invest in physical currency (at an interest rate of zero) has on yield curves, leaving a hypothetical “shadow yield curve” that would exist if physical currency were not available. The process allows one to answer the question: “what policy rate would generate the observed yield curve if the policy rate could be taken negative?”

That “shadow policy rate” therefore provides a gauge of the monetary policy stance after the actual policy rate reaches zero. For example, figure 1 shows that the shadow U.S. Federal Funds Rate (FFR) became negative from late-2008, and moved even more negative in mid-2010, both consistent with unconventional policy easings at those times.¹

Figure 1

Monthly “shadow policy rates” for the U.S. and New Zealand

Falls in the shadow policy rate provide a convenient metric for monetary policy once the ZLB becomes constraining, but such falls do not provide the same amount of economic stimulus as cuts in the actual policy rate from unconstrained levels. So in figure 1, the economic stimulus

¹ The actual FFR remained unchanged for the entire period.
associated with the shadow FFR moving from 5 percent to zero up to late-2008 was almost certainly greater than the stimulus from the unconventional monetary policy easings as represented by the shadow FFR moving from zero to -5 percent.

The ZLB constraint has not affected New Zealand’s yield curve. However, large negative shadow policy rates for the U.S. appear to have been an influence on New Zealand’s exchange rate. Therefore, the ZLB yield curve model may still be relevant, albeit indirectly, for considering New Zealand’s overall monetary policy stance.

Introduction

Operating monetary policy when interest rates are already at or near zero comes with many challenges, as various countries have discovered in recent years. Not least, if effective easing beyond a zero policy rate is desired, a policy rate that is constrained at zero will no longer summarise the stance of monetary policy and its typical transmission into the yield curve (longer-maturity interest rates), the economy, and the exchange rate.

In such environments, I show how a ZLB yield curve framework and its associated “shadow yield curve” may be applied so that the stance and transmission of monetary policy can still be considered from the perspective of an effective policy rate.

Section 1 outlines the framework and its intuition, while sections 2, 3, and 4 illustrate three perspectives on its application.

The intuition of the ZLB yield curve framework

It is well-accepted that interest rates for any time to maturity cannot fall materially below zero.\(^2\) That constraint exists because the availability of physical currency effectively offers a risk-free investment at a zero rate of interest, which will therefore be more attractive than securities offering a negative interest rate.

I use bond option pricing techniques to formally model the value of the option for investors to hold physical currency at the ZLB. That enables an estimated “ZLB/currency option effect” to be removed from the observed yield curve data, leaving the “shadow yield curve”; i.e. a hypothetical yield curve that would exist if physical currency was not available. Full details are

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\(^2\) The occasional minor exceptions are due to institutional features, such as the overhead costs of holding and transacting in physical currency, market liquidity, idiosyncratic supply and demand for specific interest rate securities, etc., and periods where high-quality fixed interest securities are purchased times of market turbulence. Switzerland, Japan, the U.S., Germany, Sweden, and Switzerland are examples of countries that have realised slightly negative interest rates for short-maturity securities, historically and/or at present.
Figure 2 illustrates the concept with an example deliberately designed to give some negative shadow interest rates. (I have set the shadow policy rate to -5.00 percent, with long-maturity yields set to level out at 5.00 percent.)

The ZLB yield curve in figure 2 is like the yield curve data we observe in practice when interest rates are materially constrained by the ZLB. Without such a constraint, interest rates as a function of maturity typically rise with a pattern similar to the shadow yield curve, but with all rates positive. The option effect is very large for short maturities in this example, because the availability of physical currency provides a large degree of protection against the negative interest rates that investors would otherwise face. For longer maturities, shadow interest rates are already positive, but model consistency still requires an option effect to account for the probability that shadow policy rates could evolve to negative values with the passage of time. Removing the option effect from the ZLB curve leaves a shadow yield curve with negative values out to around two years in this example.

**Figure 2**

*A ZLB yield curve example*

When fitted to actual interest rate data, the ZLB yield curve and its associated shadow yield curve provide unique quantitative information (albeit necessarily implicit and model dependent) useful for a variety of monetary policy purposes, as discussed in the following sections.

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3. Black (1995) originally introduced a similar framework to prevent yield curve models violating the ZLB constraint, but its application has been limited due to its computational complexity. My subtly different ZLB framework has similar results to the Black framework in practice but is easier to implement.

4. That effect is analogous to a call option with a future expiry date having a positive value even when the security on which it is written is currently above the strike price.
Summarizing the stance of monetary policy

One application of the ZLB yield curve framework is to provide a “monetary policy metric”; i.e. it allows the stance of monetary policy to be summarised as a shadow policy rate after the actual policy rate has reached the ZLB.

To illustrate, figure 3 plots the estimates of the U.S. shadow FFR obtained by estimating the ZLB yield curve model for each month-end observation of U.S. yield curve data from December 1986. The results indicate that the U.S. yield curve first became materially constrained by the ZLB in November 2008; i.e. at that time the estimated shadow FFR became negative and distinctly different from the 3-month Treasury bill rate (and the actual FFR; not plotted for clarity). Before then, the shadow FFR was not materially constrained by the ZLB, given that the shadow FFR was always positive and close to the 3-month Treasury bill rate (and the actual FFR).

Figure 3

Full sample of monthly shadow policy rates for the U.S. and New Zealand

The initial negative value and largest subsequent falls in the shadow FFR were associated with the unconventional monetary policy easing events as follows:5

- November 2008: FOMC announces QE1 (Federal Reserve purchases of mainly mortgage-backed agency securities, ultimately totalling $1,750 billion, and some other measures).
- August 2010: FOMC warns about a slowing economic recovery and Chairman Bernanke hints at QE2 (subsequently announced in November 2010, being a programme where the Federal Reserve purchased $600 billion of U.S. Treasury securities).
- 2011: FOMC warns about a slowing economic recovery (June), conditionally commits to keep the FFR exceptionally low until mid-2013 (August), and announces a maturity extension programme for the Federal Reserve’s balance sheet (September, being a switch

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5 Locating the precise timing for this and the bullet points below is easier by referring to the U.S. data markers in figure 1. Note that the vertical gridlines mark the end of December for the indicated years.
The consistency of movements in the shadow FFR and the events above indicates that, analogous to the actual FFR in conventional monetary policy environments, the shadow FFR provides a gauge of the US monetary policy stance in unconventional monetary policy environments. That said, the shadow FFR has become much more volatile since the onset of the QE2 event in August 2010, so it is more appropriate (at least for now) to use the average of around -5 percent as a broad indicator since that time. Recent estimates of the shadow FFR have also been around -5 percent, and a completely different approach summarised in Williams (2011) produces a similar value.6

Broadly then, the cumulative effect of unconventional monetary policy easing events in the US since the onset of the GFC may be represented as a decline in the shadow FFR of around 5 percentage points, from zero to -5 percent. A related interpretation is that the unconventional monetary policy easing events in the US from November 2008 in conjunction with the prevailing economic environment has led the market to expect (as at July 2012) the actual FFR will likely remain at zero for around the next 2 to 4 years.7

Unlike the U.S., New Zealand yield curve data has never been materially constrained by the ZLB over the entire available history (from March 1992, which marks the start of a standardised database – see Krippner and Thorsrud (2009) for details). The lack of any ZLB constraint is evident from the 3-month Treasury bill rate (and the actual OCR; not plotted for clarity) always being well above zero, and close to the estimated shadow OCR. That result means that all historical and current interest rate data in New Zealand can be used and interpreted without requiring any correction for the ZLB constraint.8

**Representing monetary policy stimulus**

While the shadow policy rate conveniently summarises the level and changes in the stance of monetary policy, the economic stimulus from lowering the shadow policy rate from a positive level (conventional easing) is greater than when lowering it from a negative level (unconventional easing). The difference arises, intuitively, because the declines in actual

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6 The approach estimates the falls in U.S. Treasury yields associated with QE volumes and then converts those movements into shadow FFR terms using historical estimates of FFR and Treasury yield co-movements.

7 The estimate is based on the implied path of the shadow policy rate underlying the illustrative example in figure 2 that is based on a -5 percent shadow policy rate. More precise estimates would require the full estimation of the U.S. model parameters, which is work in progress. Krippner (2012b) contains shadow policy rates and “zero horizon” estimates for Japan that range from 5 to 8 years, but because they based on a different model specification for the shadow yield curve, they are not directly comparable with the U.S. results referred to in this note.

8 Separate from ZLB considerations, risk premia for the NZ yield curve are likely to have increased from pre-GFC to post-GFC. Therefore, term interest rates are more likely to overstate the expected average path of the OCR than in the past.
interest rates (i.e. the rates relevant to the economy) along the yield curve are necessarily smaller when a negative shadow policy rate already implies a material constraint from the ZLB.

For example, figure 4 shows that the period of conventional monetary policy easing in the US from December 2006 to November 2008 (around five percentage points in the actual and shadow FFR) was accompanied by large declines in interest rates for all maturities, but particularly those with maturities up to five years. Conversely, the period of unconventional monetary policy easing from November 2008 to July 2012 (around five percentage points in the shadow FFR, from zero to -5 percent) was accompanied by much smaller declines in interest rates for maturities up to five years. Therefore, borrowers out to terms of five years received a substantial cut to their interest rates in the first period of easing, but not much in the second period. (Those funding for 7-year to 20-year horizons received similar interest rate cuts in both periods of easing, but that represents only a sub-set of borrowers.)

**Figure 4**
*Actual U.S. yield curve changes*

Figure 5 provides an illustrative example of how the yield curve changes in figure 4 can be represented within the ZLB yield curve framework. In this example, both periods of easing are deliberately designed to produce identical falls in the shadow term structure across all maturities, with the shadow policy rate falling by 5.00 percent.⁹

In the period when the shadow FFR is initially positive (i.e. a conventional monetary policy environment) the fall in the ZLB yield curve is very similar to the shadow yield curve. Conversely, in the period when the initial shadow FFR is zero (i.e. an unconventional monetary policy environment), falls in the short- and mid-maturity yields of the ZLB yield curve are muted.

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⁹ The change in the shadow yield curve is obtained with a five percentage point fall in the Slope component of the shadow yield curve, and an unchanged Level component. Discussion on these components is available in Krippner (2012a).
Figure 5
ZLB yield curve changes with positive and negative shadow policy rates

Hence, the ZLB yield curve framework conveniently establishes how movements in the actual yield curve, and the associated economic stimulus, vary as a function of the initial value of the shadow policy rate. In particular, depending on the funding maturities inherent in the economy, easings in conventional monetary policy environments likely produce substantially more economic stimulus than easings in the unconventional environments.

The exchange rate channel of monetary policy
The exchange rate is an important channel for monetary policy in small open economies such as New Zealand. Hence, this final section discusses how the ZLB yield curve framework can help to interpret the level and movements in New Zealand’s exchange rate.

Referring to figure 6, the NZ/US exchange rate has typically moved broadly in line with the 3-month Treasury bill rate differential in the past. However, from late-2008, when the U.S. adopted a zero interest rate policy, a gap opened up relative to the usual historical relationship.
Using the shadow policy rate differential rather than the 3-month Treasury bill differential maintains a more stable relationship with the NZ/US exchange rate over the entire period.\textsuperscript{10} Specifically, the shadow policy rate differential has widened substantially since late-2008, and the appreciation of the NZ/US exchange rate from that time is consistent with the historical relationship prior to late-2008. In economic terms, both the currency appreciation and the widening interest rate differential probably reflect relatively better perceptions of the state of the New Zealand economy relative to the U.S. in recent times.\textsuperscript{11}

**Conclusion**

This note outlines a ZLB yield curve framework that can be applied to several aspects relevant to the operation of monetary policy in both conventional and unconventional environments. For example, the “shadow policy rate” in the framework continues to provide a gauge of the monetary policy stance after the actual policy rate reaches the ZLB, and the framework also represents how changes in the actual yield curve from policy easings become muted once the ZLB becomes constraining. The historical relationship between the NZ/US exchange rate and short-term interest rate differentials is also better maintained using shadow policy rates.

\textsuperscript{10} Alternatively, the relationship could also be maintained by shifting to 10-year interest rate differentials, where the effect of the ZLB is much less material than for shorter-maturity interest rates. However, the shadow short rate differential is more comprehensive because it effectively condenses the information from the entire term structure into a single point.

\textsuperscript{11} McDonald (2012) finds that interest rate differentials are not substantial explanatory variables for the exchange rate in general, relative to the explanatory power of commodity prices, etc. However, the point remains that the marginal explanatory power could be improved with a shadow interest rate differential.
References


Krippner, L. (2012a), "Modifying Gaussian term structure models when interest rates are near the zero lower bound", Discussion Paper 2012/02, Reserve Bank of New Zealand.


